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Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section

P. Ponnusamy *

Department of Mathematics, Government Arts College (Autonomous), Coimbatore, Tamil Nadu 641 018, India

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Abstract

In this article, the wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section is discussed, using the Fourier expansion collocation method. The solid medium is assumed to be linear, isotropic, and dependent on the rate of temperature. Three displacement potential functions are introduced, to uncouple the equations of motion and the heat conduction. By imposing the continuity conditions the frequency equation corresponding to the problem is obtained using the Fourier expansion collocation method based on Suhubi's generalized theory [Suhubi, E.S., 1975. Thermoelastic Solids. In: Eringen, A.C. (Ed.), Continuum Physics, vol. 2. Academic, New York, Chapter 2]. To compare the model with the existing literature, the results of a generalized thermoelastic solid cylinder are obtained and they are compared with the results of Erbay and Suhubi [Erbay, E.S., Suhubi, E.S., 1986. Longitudinal wavepropagationed thermoelastic cylinder. J. Thermal Stresses 9, 279–295]. It shows very good degree of agreement. The computed non-dimensional wavenumbers are presented in figures for various values of the material parameters. The general theory can be used to study any kind of cylinders with proper geometrical relations.

Keywords: Wave propagation in isotropic cylinders of arbitrary cross-sections; Free vibration of transversely isotropic cylinder; Elastic rods loaded with fluid; Wave propagation in rod; Wave propagation in plates; Vibration of thermoelastic rod; Vibration generalized thermoelastic cylinder

1. Introduction

The propagation of waves in thermoelastic materials has many applications in various fields of science and technology, namely, atomic physics, industrial engineering, thermal power plants, submarine structures, pressure vessel, aerospace, chemical pipes, and metallurgy. The importance of thermal stresses in causing structural damages and changes in functioning of the structure is well recognized whenever thermal stress environments are involved. Therefore, the ability to predict electrodynamics stress induced by sudden thermal loading in composite structures is essential for the proper and safe design and the knowledge of its response during the service in these severe thermal environments.

* Tel.: +91 94431 91638.
E-mail address: ponnusamypp@yahoo.com

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A method, for solving wave propagation in arbitrary cross-sectional cylinders and plates and to find out the phase velocities in different modes of vibrations namely longitudinal, torsional, and flexural, by constructing frequency equations was devised (Nagaya, 1982, 1983, 1984, 1985). He formulated the Fourier expansion collocation method for this purpose. Following Nagaya, Paul and Venkatesan (1987) studied the wave propagation in an infinite piezoelectric solid cylinder of arbitrary cross-section using Fourier expansion collocation method.

The generalized theory of thermoelasticity was developed by Lord and Shulman (1967) involving one relaxation time for isotropic homogeneous media, which is called the first generalization to the coupled theory of elasticity. These equations determine the finite speeds of propagation of heat and displacement distributions, the corresponding equations for an isotropic case were obtained by Dhaliwal and Sherief (1980).

The second generalization to the coupled theory of elasticity is what is known as the theory of thermoelasticity, with two relaxation times or the theory of temperature-dependent theomoelasticity. A generalization of this inequality was proposed by Green and Laws (1972). Green and Lindsay (1972) obtained an explicit version of the constitutive equations. These equations were also obtained independently by Suhubi (1964). This theory contains two constants that act as relaxation times and modify not only the heat equations, but also all the equations of the coupled theory. The classical Fourier's law of heat conduction is not violated if the medium under consideration has a center of symmetry. Erbay and Suhubi (1986) studied the longitudinal wave propagation in a generalized thermoelastic infinite cylinder and obtained the dispersion relation for a constant surface temperature of the cylinder.

Singh and Sharma (1985), Sharma and Sidhu (1986) investigated the generalized thermoelastic waves in transversely isotropic and anisotropic media, respectively. Sharma (2001) discussed the three-dimensional vibration analysis of a homogeneous transversely isotropic thermoelastic cylindrical panel. Verma (2002) presented the propagation of waves in layered anisotropic media in generalized thermo elasticity in an arbitrary layered plate.

Venkatesan and Ponnusamy (2002, 2003) have obtained the frequency equation of the free vibration of a solid cylinder of arbitrary cross-section immersed in a fluid using Fourier expansion collocation method. The frequency equations are obtained for longitudinal and flexural vibrations and are studied numerically for elliptical and cardioidal cross-sectional cylinders.

In this paper, the free vibration of a generalized thermoelastic solid cylinder of arbitrary cross-section is studied using the Fourier expansion collocation method based on Suhubi's generalized theory (1975). The frequency equations of longitudinal and flexural (symmetric and antisymmetric) modes are analyzed numerically for the material copper. The computed non-dimensional wavenumbers are plotted as a graph.

2. Formulation of the problem

We consider a homogeneous, isotropic, thermally conducting elastic solid cylinder of arbitrary cross-section with uniform temperature T_0 in the undisturbed state initially. The system displacements and stresses are defined by the cylindrical coordinates r, θ, and z. In cylindrical coordinates, the three-dimensional stress equations of motion and strain-displacement relations and heat conduction in the absence of body force for a linearly elastic medium are

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho u_{,tt} \tag{1a}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{,rzz} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt} \tag{1b}$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{r\theta} = \rho w_{,tt} \tag{1c}$$

$$K\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta} + T_{,zz}\right) = \rho c_v T_{,t} + \rho\tau T_{,tt} + \beta T_0\left[u_{,rt} + r^{-1}(u_{,t} + v_{,\theta t}) + w_{,tz}\right] \tag{1d}$$

and

$$\sigma_{rr} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{rr} - \beta(T + \eta T_{,t}) \tag{2a}$$
$$\sigma_{\theta\theta} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{\theta\theta} - \beta(T + \eta T_{,t}) \tag{2b}$$
$$\sigma_{zz} = \lambda(e_{rr} + e_{\theta\theta} + e_{zz}) + 2\mu e_{zz} - \beta(T + \eta T_{,t}) \tag{2c}$$
$$\sigma_{r\theta} = 2\mu\gamma_{r\theta} \tag{2d}$$
$$\sigma_{\theta z} = 2\mu\gamma_{\theta z} \tag{2e}$$
$$\sigma_{rz} = 2\mu\gamma_{rz} \tag{2f}$$

where σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz}, $\sigma_{r\theta}$, $\sigma_{\theta z}$, and σ_{rz} are the stress components, e_{rr}, $e_{\theta\theta}$, e_{zz}, $e_{r\theta}$, $e_{\theta z}$, and e_{rz} are the strain components, T is the temperature change about the equilibrium temperature T_0, ρ is the mass density, c_v is the specific heat capacity, β is a coupling factor that couples the heat conduction and elastic field equations, K is the thermal conductivity, τ, η are the thermal relaxation times, t is the time, λ and μ are Lame' constants.

The strain e_{ij} are related to the displacements are given by

$$e_{rr} = u_{,r}, \quad e_{\theta\theta} = r^{-1}(u + v_{,\theta}), \quad e_{zz} = w_{,z}, \tag{3a}$$
$$\gamma_{r\theta} = v_{,r} - r^{-1}(v - u_{,\theta}), \quad \gamma_{z\theta} = v_{,z} + r^{-1}w_{,\theta}, \quad \gamma_{rz} = w_{,r} + u_{,z} \tag{3b}$$

in which u, v, and w are the displacement components along radial, circumferential, and axial directions, respectively. The comma in the subscripts denotes the partial differentiation with respect to the variables.

Substituting the Eqs. (3) and (2) in Eq. (1), the following displacement equations of motion are obtained as

$$\begin{aligned}(\lambda + 2\mu)\left(u_{,rr} + r^{-1}u_{,r} - r^{-2}u\right) + \mu r^{-2}u_{,\theta\theta} + \mu u_{,zz} + r^{-1}(\lambda + \mu)v_{,r\theta} \\ - r^{-2}(\lambda + 3\mu)v_{,\theta} + (\lambda + \mu)w_{,rz} - \beta(T_{,r} + \eta T_{,rt}) = \rho u_{,tt}\end{aligned} \tag{4a}$$
$$\begin{aligned}\mu\left(v_{,rr} + r^{-1}v_{,r} - r^{-2}v\right) + r^{-2}(\lambda + 2\mu)v_{,\theta\theta} + \mu v_{,zz} + r^{-2}(\lambda + 3\mu)u_{,\theta} \\ + r^{-1}(\lambda + \mu)u_{,r\theta} + r^{-1}(\lambda + \mu)w_{,\theta z} - \beta(T_{,\theta} + \eta T_{,\theta t}) = \rho v_{,tt}\end{aligned} \tag{4b}$$
$$\begin{aligned}(\lambda + 2\mu)w_{,zz} + \mu\left(w_{,rr} + r^{-1}w_{,r} + r^{-2}w_{,\theta\theta}\right) + (\lambda + \mu)u_{,rz} + r^{-1}(\lambda + \mu)v_{,\theta z} \\ + r^{-1}(\lambda + \mu)u_{,z} - \beta(T_{,z} + \eta T_{,zt}) = \rho w_{,tt}\end{aligned} \tag{4c}$$
$$\rho c_v \kappa\left(T_{,rr} + r^{-1}T_{,r} + r^{-2}T_{,\theta\theta} + T_{,zz}\right) = \rho\tau T_{,tt} + \rho c_v T_{,t} + \beta T_0\left[u_{,tr} + r^{-1}(u_{,t} + v_{,t\theta}) + w_{,tz}\right] \tag{4d}$$

$\kappa = K/\rho c_v$ is the diffusivity.

3. Solution of the equations

The Eq. (4) is a coupled partial differential equations of the three displacement and heat conduction components. To uncouple the Eq. (4), we follow Mirsky (1964) and assuming the solution of Eq. (4) as follows:

$$u(r,\theta,z,t) = \sum_{n=0}^{\infty} \varepsilon_n\left[\left(\phi_{n,r} + r^{-1}\psi_{n,\theta}\right) + \left(\overline{\phi}_{n,r} + r^{-1}\overline{\psi}_{n,\theta}\right)\right]e^{i(kz+\omega t)} \tag{5a}$$
$$v(r,\theta,z,t) = \sum_{n=0}^{\infty} \varepsilon_n\left[\left(r^{-1}\phi_{n,\theta} - \psi_{n,r}\right) + \left(r^{-1}\overline{\phi}_{n,\theta} - \overline{\psi}_{n,r}\right)\right]e^{i(kz+\omega t)} \tag{5b}$$
$$w(r,\theta,z,t) = (i/a)\sum_{n=0}^{\infty} \varepsilon_n\left[W_n + \overline{W}_n\right]e^{i(kz+\omega t)} \tag{5c}$$
$$T(r,\theta,z,t) = \left((\lambda + 2\mu)/\beta a^2\right)\sum_{n=0}^{\infty}\left[T_n + \overline{T}_n\right]e^{i(kz+\omega t)} \tag{5d}$$

where $\varepsilon_n = \frac{1}{2}$ for $n = 0$, $\varepsilon_n = 1$ for $n \geqslant 1$, $i = \sqrt{-1}$, k is the wavenumber, ω is the frequency, $\phi_n(r,\theta)$, $W_n(r,\theta)$, $T_n(r,\theta)$, $\psi_n(r,\theta)$, $\overline{\phi}_n(r,\theta)$, $\overline{W}_n(r,\theta)$, $\overline{T}_n(r,\theta)$, $\overline{\psi}_n(r,\theta)$ are the displacement potentials, and a is the geometrical parameter of the cylinder.

Introducing the irrotational velocity $c_1^2 = (\lambda + 2\mu)/\rho$ and dimensionless quantities such as $\varsigma = ka$, $\bar{z} = z/a$, $T_a = t\sqrt{\mu/\rho}/a$, $x = r/a$, $\alpha' = c_1 a/\kappa$, $\Omega^2 = \omega^2 a^2/c_1^2$, $\chi_1 = \frac{T_0 a}{\rho^2 c_v c_1 \kappa}\beta^2$, $\chi_2 = \frac{c_1^2}{c_v \kappa}\tau$, $\chi_3 = \frac{c_1}{a}\eta$, and $\chi_4 = 1/(2+\bar{\lambda})$ and using Eq. (5) in Eq. (4), we obtain

$$\left(\nabla^2 + \Omega^2 - \chi_4\varsigma\right)\phi_n - \varsigma\left(1+\bar{\lambda}\right)\chi_4 W_n - (1 + \mathrm{i}\chi_3\Omega)T_n = 0 \tag{6a}$$

$$\varsigma\left(1+\bar{\lambda}\right)\chi_4\nabla^2\phi_n + \left(\chi_4\nabla^2 + \Omega^2 - \varsigma^2\right)W_n - \varsigma(1 + \mathrm{i}\chi_3\Omega)T_n = 0 \tag{6b}$$

$$-\mathrm{i}\chi_1\Omega\nabla^2\phi_n + \mathrm{i}\chi_1\varsigma\Omega W_n + \left(\nabla^2 - \mathrm{i}\alpha'\Omega + \chi_2\right)T_n = 0 \tag{6c}$$

and

$$\left(\nabla^2 + \left(2+\bar{\lambda}\right)\Omega^2 - \varsigma^2\right)\psi_n = 0 \tag{7}$$

where $\nabla^2 \equiv \partial^2/\partial x^2 + x^{-1}\partial/\partial x + x^{-2}\partial^2/\partial\theta^2$.

The parameters defined in Eq. (6) namely, χ_1 couples the equations corresponding to the elastic wave propagation and the heat conduction which is called the coupling factor; the coefficient χ_2, which is introduced by the theory of generalized thermoelasticity, may render the governing system of equations hyperbolic. The parameter χ_3 is the coefficient of the term indicating the difference between empirical and thermodynamic temperatures.

Rewriting Eq. (6), results in the following equation:

$$\begin{vmatrix} \left(\nabla^2 + \Omega^2 - \chi_4\varsigma^2\right) & -\varsigma\chi_4\left(1+\bar{\lambda}\right) & (1 + \mathrm{i}\chi_3\Omega) \\ \varsigma\chi_4\left(1+\bar{\lambda}\right) & \left(\nabla^2\chi_4 + \Omega^2 - \varsigma^2\right) & -\varsigma(1 + \mathrm{i}\chi_3\Omega) \\ -\mathrm{i}\chi_1\Omega & \mathrm{i}\varsigma\chi_1\Omega & \left(\nabla^2 - \mathrm{i}\alpha'\Omega + \chi_2\right) \end{vmatrix}(\phi_n, W_n, T_n) = 0. \tag{8}$$

Eq. (8), on simplification reduces to the following differential equation:

$$\left(A\nabla^6 + B\nabla^4 + C\nabla^2 + D\right)(\phi_n, W_n, T_n) = 0 \tag{9}$$

where

$$A = \chi_4 \tag{10a}$$

$$B = \chi_4\left(\Omega^2 - \chi_4\varsigma^2 + \chi_2 - \mathrm{i}\alpha\Omega\right) + \varsigma^2\chi_4^2\left(1+\bar{\lambda}\right)^2 + \left(\Omega^2 - \varsigma^2\right) \tag{10b}$$

$$\begin{aligned} C = {} & \mathrm{i}\varsigma\Omega(1 + \mathrm{i}\chi_3\Omega)\left(\varsigma^2 + \varsigma^2\chi_4\left(1+\bar{\lambda}\right) + \chi_4\right) + \left(\Omega^2 - \chi_4\varsigma^2\right)\left(\Omega^2 - \varsigma^2 - \chi_4(\chi_2 - \mathrm{i}\Omega\alpha')\right) \\ & - (\chi_2 - \mathrm{i}\Omega\alpha')\left(\left(\Omega^2 - \varsigma^2\right) + \varsigma^2\chi_4^2\left(1+\bar{\lambda}\right)\right) \end{aligned} \tag{10c}$$

$$\begin{aligned} D = {} & \mathrm{i}\Omega\alpha'(1 + \mathrm{i}\chi_3\Omega)\left(-\varsigma^2\chi_4\left(1+\bar{\lambda}\right) + \left(\Omega^2 - \chi_4\varsigma^2\right)\varsigma^2 + \left(\Omega^2 - \varsigma^2\right)\right) \\ & + (\chi_2 - \mathrm{i}\Omega\alpha')\left(\Omega^2 - \chi_4\varsigma^2\right)\left(\Omega^2 - \varsigma^2\right) \end{aligned} \tag{10d}$$

Factorizing the partial differential equation given in Eq. (9) into cubic equation for $(\alpha_j a)^2$, $(j = 1, 2, 3)$, the solution for the symmetric mode are obtained as

$$\phi_n = \sum_{j=1}^{3} A_{jn}J_n\left(\alpha_j ax\right)\cos n\theta \tag{11a}$$

$$W_n = \sum_{j=1}^{3} d_j A_{jn}J_n\left(\alpha_j ax\right)\cos n\theta \tag{11b}$$

$$T_n = \sum_{j=1}^{3} e_j A_{jn}J_n\left(\alpha_j ax\right)\cos n\theta \tag{11c}$$

The solution for the antisymmetric modes $\bar{\phi}_n$, $\bar{W}_n$, and $\bar{T}_n$ are obtained by replacing $\cos n\theta$ by $\sin n\theta$ in Eq. (11). Since we are considering solid cylinder of arbitrary cross-section, the Bessel function of the second kind Y_n is absent.

Here $(\alpha_j a)^2 > 0$, $(j = 1, 2, 3)$ are the roots of the algebraic equation

$$A(\alpha a)^6 - B(\alpha a)^4 + C(\alpha a)^2 + D = 0 \tag{12}$$

The solutions corresponding to the root $(\alpha_j a)^2 = 0$ is not considered here, since $J_n(0)$ are zero, except for $n = 0$. The Bessel function J_n is used when the roots $(\alpha_j a)^2$, $(j = 1,2,3)$ are real or complex and the modified Bessel function I_n is used when the roots $(\alpha_j a)^2$, $(j = 1,2,3)$ are imaginary.

The constants d_j and e_j defined in the Eq. (11) can be calculated from the equations

$$\chi_4\varsigma(1+\bar{\lambda})d_j + (1 + \mathrm{i}\chi_3\Omega)e_j = (\Omega^2 - \varsigma^2)\chi_4 - (\alpha_j a)^2 \tag{13a}$$

$$\left((\Omega^2 - \varsigma^2) - (\alpha_j a)^2\chi_4\right)d_j - \varsigma(1 + \mathrm{i}\chi_3\Omega)e_j = (\alpha_j a)^2\chi_4\varsigma(1+\bar{\lambda}) \tag{13b}$$

Solving the Eq. (7), the solution to the symmetric mode is obtained as

$$\psi_n = A_{4n}J_n(\alpha_4 ax)\sin n\theta \tag{14}$$

where $(\alpha_4 a)^2 = \Omega^2 - \varsigma^2$. If $(\alpha_4 a)^2 < 0$, the Bessel function J_n is replaced by the modified Bessel function I_n. The solution for the antisymmetric mode $\bar{\psi}_n$ is obtained from Eq. (14) by replacing $\sin n\theta$ by $\cos n\theta$.

4. Boundary conditions and frequency equations

In this problem, the free vibration of a generalized thermoelastic solid cylinder of arbitrary cross-section is considered. Since the boundary is irregular, the Fourier expansion collocation method is applied on the boundary of the cross-section. Thus, the boundary conditions obtained are

$$(\sigma_{pp})_l = (\sigma_{pq})_l = (\sigma_{zp})_l = (T)_l = 0 \tag{15}$$

where p is the coordinate normal to the boundary and q is the coordinate in the tangential direction. Here σ_{pp} is the normal stress, σ_{pq} and σ_{zp} are the shearing stresses and $(\ )_l$ is the value at the lth segment of the boundary. Since the coordinate p and q are functions of r and θ, it is difficult to find transformed expressions for the stresses. Therefore the curved boundary is divided into small segments such that the variations of the stresses are assumed to be constant. Assuming the angle γ_l, between the normal to the segment and the reference axis to be constant, the transformed expressions for the stresses are followed by Nagaya (1982, 1984, 1985, 1983)

$$\begin{aligned}\sigma_{pp} = {} & 2\mu\left[u_{,r}\cos^2(\theta-\gamma_l) + r^{-1}(u + v_{,\theta})\sin^2(\theta-\gamma_l) + 0.5\left(r^{-1}[u - u_{,\theta}] - v_{,r}\right)\sin 2(\theta-\gamma_l)\right] \\ & + \lambda\left(u_{,r} + r^{-1}(u + v_{,\theta}) + w_{,z}\right)\end{aligned} \tag{16a}$$

$$\sigma_{pq} = \mu\left[(u_{,r} - r^{-1}(v_{,\theta} + u))\sin 2(\theta-\gamma_l) + (r^{-1}(u_{,\theta} - v) + v_{,r})\cos 2(\theta-\gamma_l)\right] \tag{16b}$$

$$\sigma_{zq} = \mu\left[(u_{,z} + w_{,r})\cos(\theta-\gamma_l) - (v_{,z} + r^{-1}w_{,\theta})\sin(\theta-\gamma_l)\right] \tag{16c}$$

Applying the Fourier expansion collocation method along the curved surface of the boundary, the transformed expressions for the stresses are

$$\left[(S_{pp})_l + (\bar{S}_{pp})_l\right]\mathrm{e}^{\mathrm{i}(\varsigma\bar{z}+\Omega T_a)} = 0 \tag{17a}$$

$$\left[(S_{pq})_l + (\bar{S}_{pq})_l\right]\mathrm{e}^{\mathrm{i}(\varsigma\bar{z}+\Omega T_a)} = 0 \tag{17b}$$

$$\left[(S_{zp})_l + (\bar{S}_{zp})_l\right]\mathrm{e}^{\mathrm{i}(\varsigma\bar{z}+\Omega T_a)} = 0 \tag{17c}$$

$$\left[(S_t)_l + (\bar{S}_t)_l\right]\mathrm{e}^{\mathrm{i}(\varsigma\bar{z}+\Omega T_a)} = 0 \tag{17d}$$

where,

$$S_{pp} = 0.5\left(A_{10}e_0^1 + A_{20}e_0^2 + A_{30}e_0^3 + B_{50}e_0^5\right) + \sum_{n=1}^{\infty}\left(A_{1n}e_n^1 + A_{2n}e_n^2 + A_{3n}e_n^3 + A_{4n}e_n^4 + B_{5n}e_n^5\right) \tag{18a}$$

$$S_{pq} = 0.5\left(A_{10}f_0^1 + A_{20}f_0^2 + A_{30}f_0^3\right) + \sum_{n=1}^{\infty}\left(A_{1n}f_n^1 + A_{2n}f_n^2 + A_{3n}f_n^3 + A_{4n}f_n^4\right) \tag{18b}$$

$$S_{zp} = 0.5\left(A_{10}g_0^1 + A_{20}g_0^2 + A_{30}g_0^3\right) + \sum_{n=1}^{\infty}\left(A_{1n}f_n^1 + A_{2n}f_n^2 + A_{3n}f_n^3 + A_{4n}f_n^4\right) \tag{18c}$$

$$S_t = 0.5\left(A_{10}k_0^1 + A_{20}k_0^2 + A_{30}k_0^3\right) + \sum_{n=1}^{\infty}\left(A_{1n}k_n^1 + A_{2n}k_n^2 + A_{3n}k_n^3\right) \tag{18d}$$

$$\overline{S}_{pp} = 0.5\overline{A}_{40}\bar{e}_0^4 + \sum_{n=1}^{\infty}\left(\overline{A}_{1n}\bar{e}_n^1 + \overline{A}_{2n}\bar{e}_n^2 + \overline{A}_{3n}\bar{e}_n^3 + \overline{A}_{4n}\bar{e}_n^4 + \overline{B}_{5n}\bar{e}_n^5\right) \tag{19a}$$

$$\overline{S}_{pq} = 0.5\overline{A}_{40}\bar{f}_0^4 + \sum_{n=1}^{\infty}\left(\overline{A}_{1n}\bar{f}_n^1 + \overline{A}_{2n}\bar{f}_n^2 + \overline{A}_{3n}\bar{f}_n^3 + \overline{A}_{4n}\bar{f}_n^4\right) \tag{19b}$$

$$\overline{S}_{zp} = 0.5\overline{A}_{40}\bar{g}_0^4 + \sum_{n=1}^{\infty}\left(\overline{A}_{1n}\bar{g}_n^1 + \overline{A}_{2n}\bar{g}_n^2 + \overline{A}_{3n}\bar{g}_n^3 + \overline{A}_{4n}\bar{g}_n^4\right) \tag{19c}$$

$$\overline{S}_t = \sum_{n=1}^{\infty}\left(\overline{A}_{1n}\bar{k}_n^1 + \overline{A}_{2n}\bar{k}_n^2 + \overline{A}_{3n}\bar{k}_n^3\right) \tag{19d}$$

The functions $e_n^j \sim \bar{k}_n^j$ used in the boundary conditions of the symmetric and antisymmetric cases are given in Appendix A.

The boundary conditions along the entire range of the boundary cannot be satisfied directly. To satisfy the boundary conditions, the Fourier expansion collocation method due to Nagaya (1982, 1984, 1985, 1983) is applied along the boundary. Performing the Fourier series expansion to the transformed expression in Eq. (15) along the boundary, the boundary conditions are expanded in the form of double Fourier series for symmetric and antisymmetric modes of vibrations. For the symmetric mode, the equation, which satisfies the boundary conditions, is obtained in matrix form as follows:

$$\begin{bmatrix}
E_{00}^1 & E_{00}^2 & E_{00}^3 & E_{01}^1 & \cdots & E_{0N}^1 & E_{01}^2 & \cdots & E_{0N}^2 & E_{01}^3 & \cdots & E_{0N}^3 & E_{01}^4 & \cdots & E_{0N}^4 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
E_{N0}^1 & E_{N0}^2 & E_{N0}^3 & E_{N1}^1 & \cdots & E_{NN}^1 & E_{N1}^2 & \cdots & E_{NN}^2 & E_{N1}^3 & \cdots & E_{NN}^3 & E_{N1}^4 & \cdots & E_{NN}^4 \\
F_{10}^1 & F_{10}^2 & F_{10}^3 & F_{11}^1 & \cdots & F_{1N}^1 & F_{11}^2 & \cdots & F_{1N}^2 & F_{11}^3 & \cdots & F_{1N}^3 & F_{11}^4 & \cdots & F_{1N}^4 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
F_{N0}^1 & F_{N0}^2 & F_{N0}^3 & F_{N1}^1 & \cdots & F_{NN}^1 & F_{N1}^2 & \cdots & F_{NN}^2 & F_{N1}^3 & \cdots & F_{NN}^3 & F_{N1}^4 & \cdots & F_{NN}^4 \\
G_{00}^1 & G_{00}^2 & G_{00}^4 & G_{01}^1 & \cdots & G_{0N}^1 & G_{01}^2 & \cdots & G_{0N}^2 & G_{01}^3 & \cdots & G_{0N}^3 & G_{01}^4 & \cdots & G_{0N}^4 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
G_{N0}^1 & G_{N0}^2 & G_{N0}^3 & G_{N1}^1 & \cdots & G_{NN}^1 & G_{N1}^2 & \cdots & G_{NN}^2 & G_{N1}^3 & \cdots & G_{NN}^3 & G_{N1}^4 & \cdots & G_{NN}^4 \\
K_{00}^1 & K_{00}^2 & K_{00}^3 & K_{01}^1 & \cdots & K_{0N}^1 & K_{01}^2 & \cdots & K_{0N}^2 & K_{01}^3 & \cdots & K_{0N}^3 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
K_{N0}^1 & K_{N0}^2 & K_{N0}^3 & K_{N1}^1 & \cdots & K_{NN}^1 & K_{N1}^2 & \cdots & K_{NN}^2 & K_{N1}^3 & \cdots & K_{NN}^3 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
A_{10} \\ A_{20} \\ A_{30} \\ A_{11} \\ \vdots \\ A_{1N} \\ \vdots \\ \vdots \\ \vdots \\ A_{41} \vdots \\ \vdots \\ A_{4N}
\end{bmatrix} = 0 \tag{20}$$

where,

$$E^j_{mn} = (2\varepsilon_n/\pi)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} e^j_n(R_l,\theta)\cos m\theta\,\mathrm{d}\theta \tag{21a}$$

$$F^j_{mn} = (2\varepsilon_n/\pi)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} f^j_n(R_l,\theta)\sin m\theta\,\mathrm{d}\theta \tag{21b}$$

$$G^j_{mn} = (2\varepsilon_n/\pi)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} g^j_n(R_l,\theta)\cos m\theta\,\mathrm{d}\theta \tag{21c}$$

$$K^j_{mn} = (2\varepsilon_n/\pi)\sum_{l=1}^{L}\int_{\theta_{l-1}}^{\theta_l} k^j_n(R_l,\theta)\cos m\theta\,\mathrm{d}\theta \tag{21d}$$

Here $j = 1,2,3,4$ and 5, L is the number of segments, R_l is the coordinate r at the boundary and N is the number of terms in the Fourier series.

The boundary conditions for the antisymmetric mode are written in the form of a matrix as given below:

$$\begin{bmatrix}
\overline{E}^4_{10} & \overline{E}^1_{11} & \cdots & \overline{E}^1_{1N} & \overline{E}^2_{11} & \cdots & \overline{E}^2_{1N} & \overline{E}^3_{11} & \cdots & \overline{E}^3_{1N} & \overline{E}^4_{11} & \cdots & \overline{E}^4_{1N} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{E}^4_{N0} & \overline{E}^1_{N1} & \cdots & \overline{E}^1_{NN} & \overline{E}^2_{N1} & \cdots & \overline{E}^2_{NN} & \overline{E}^3_{N1} & \cdots & \overline{E}^3_{NN} & \overline{E}^4_{N1} & \cdots & \overline{E}^4_{NN} \\
\overline{F}^3_{00} & \overline{F}^1_{01} & \cdots & \overline{F}^1_{0N} & \overline{F}^2_{01} & \cdots & \overline{F}^2_{0N} & \overline{F}^3_{01} & \cdots & \overline{F}^3_{0N} & \overline{F}^4_{01} & \cdots & \overline{F}^4_{0N} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{F}^4_{N0} & \overline{F}^1_{N1} & \cdots & \overline{F}^1_{NN} & \overline{F}^2_{N1} & \cdots & \overline{F}^2_{NN} & \overline{F}^3_{N1} & \cdots & \overline{F}^3_{NN} & \overline{F}^4_{N1} & \cdots & \overline{F}^4_{NN} \\
\overline{G}^4_{10} & \overline{G}^1_{11} & \cdots & \overline{G}^1_{1N} & \overline{G}^2_{11} & \cdots & \overline{G}^2_{1N} & \overline{G}^3_{11} & \cdots & \overline{G}^3_{1N} & \overline{G}^4_{11} & \cdots & \overline{G}^4_{1N} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{G}^4_{N0} & \overline{G}^1_{N1} & \cdots & \overline{G}^1_{NN} & \overline{G}^2_{N1} & \cdots & \overline{G}^2_{NN} & \overline{G}^3_{N1} & \cdots & \overline{G}^3_{NN} & \overline{G}^4_{N1} & \cdots & \overline{G}^4_{NN} \\
\overline{K}^4_{10} & \overline{K}^1_{11} & \cdots & \overline{K}^1_{1N} & \overline{K}^2_{11} & \cdots & \overline{K}^2_{1N} & \overline{K}^3_{11} & \cdots & \overline{K}^3_{1N} & 0 & \cdots & 0 \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\overline{K}^4_{N0} & \overline{K}^1_{N1} & \cdots & \overline{K}^1_{NN} & \overline{K}^2_{N1} & \cdots & \overline{K}^2_{NN} & \overline{K}^3_{N1} & \cdots & \overline{K}^3_{NN} & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
\overline{A}_{40} \\ \overline{A}_{11} \\ \vdots \\ \overline{A}_{1N} \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \overline{A}_{41} \\ \vdots \\ \overline{A}_{4N}
\end{bmatrix} = 0 \tag{22}$$

The Fourier coefficients $\overline{E}^j_n$, $\overline{F}^j_n$, $\overline{G}^j_n$, and $\overline{K}^j_n$, are obtained by replacing $\cos m\theta$ by $\sin m\theta$ and $\sin m\theta$ by $\cos m\theta$ in Eq. (21). For the non-trivial solution of the systems of equations, given in Eqs. (20) and (22), the determinant of the coefficient matrix must vanish and these determinants give the frequencies of symmetric and antisymmetric modes of vibrations, respectively.

5. Numerical results and discussion

In accordance with the theoretical results obtained in the previous sections and comparing these results with the literature results, few numerical analysis of the dispersion equation is carried out for elliptic and cardioidal cross-sectional cylinders. The secant method given by Antia (2002) is used to obtain the roots of the frequency equation. The material properties of copper at temperature 4.2 K are taken approximately as Poisson ratio $\nu = 0.3$, the Young's modulus $E = 2.139\times10^{11}\ \mathrm{N/m^2}$, $\lambda = 8.20\times10^{11}\ \mathrm{kg/ms^2}$, $\mu = 4.20\times10^{10}\ \mathrm{kg/ms^2}$, $c_v = 9.1\times10^{-2}\ \mathrm{m^2/ks^2}$, $K = 113\times10^{2}\ \mathrm{kg\,m/ks^2}$, and $\rho = 8.96\times10^{3}\ \mathrm{kg/m^3}$. The other parameters such as α', χ_1, χ_2, and χ_3 are chosen by following the arguments given by Erbay and Suhubi (1986).

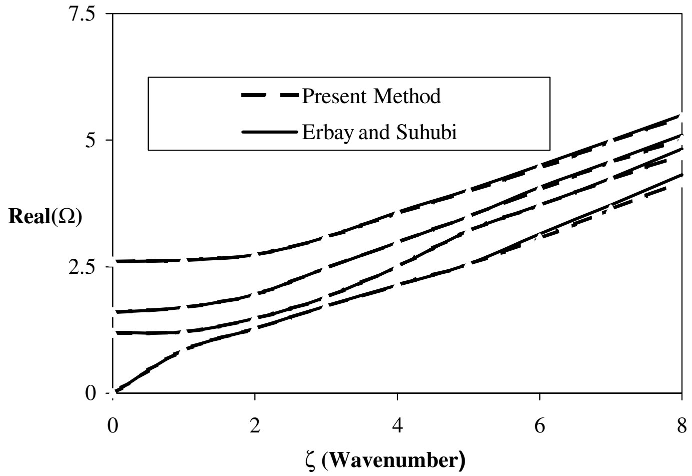

Fig. 1. Comparison between present method with Erbay and Suhubi (1986). Non-dimensional wavenumber versus real part of non-dimensional frequency Ω for circular copper cylinder for $\alpha' = 2.0$, $\chi_1 = \chi_2 = \chi_3 = 1.0$.

5.1. Longitudinal mode

The geometrical relation for elliptic cross-sections is taken from Eq. (11) of Nagaya (1982), and they are used directly for the numerical calculation. Since the elliptic cross-section vibrates along the axis of the cylinder, the vibrational displacements are symmetrical about both major and minor axes. Therefore n and m are chosen as $0, 2, 4, 6, \ldots$ in Eq. (20) for numerical calculation. The elliptical curve in the range $\theta = 0$ and $\theta = \pi$ is divided into 20 segments, such that the distance between any two segments is negligible. To illustrate the validity of the model, the frequency equation is first solved with $a/b = 1.0$, $\alpha' = 2.0$, $\chi_1 = \chi_2 = \chi_3 = 1.0$ and the dimensionless frequency $0 \leqslant \Omega \leqslant 1.0$ is obtained by fixing the dimensionless wavenumber ς. A graph is drawn from these results by taking ς along the x-axis and the real part of Ω along the y-axis. The graph of the present method, indicated by the dotted lines is compared with (Fig. 7) that of Erbay and Suhubi (1986), represented by solid lines is shown in Fig. 1. It shows perfect matching between the two. This validates the present method, thereby, numerical analysis is continued further to obtain the dimensionless wavenumber of an elliptic and cardioidal cylinders with different thermal parameters.

5.2. Flexural mode

In the case of flexural mode of elliptical cross-section, the vibration displacements are antisymmetrical about the major axis and symmetrical about the minor axis. Hence the frequency equation is obtained from the Eq. (22) by choosing both the terms as $n, m = 1, 3, 5 \ldots$ Two kinds of flexural (symmetric and antisymmetric) modes are considered. The dispersion curves are drawn for different material parameters and they are presented.

The same problem is solved for a cardioidal cross-sectional cylinder using the geometrical relations given in Eqs. (24) and (26) of Nagaya (1983), which are functions depending on a parameter s. Since a cardioid is symmetric about only one axis, the longitudinal and flexural symmetric modes are carried out by choosing $n, m = 0, 1, 2, 3 \ldots$ in Eq. (20) and flexural (antisymmetric) modes are obtained by choosing $n, m = 1, 2, 3 \ldots$ in Eq. (22). This parameter s represents a circle when $s = 0$ and represents a cardioid when $s = 0.5$. First the parameter $s = 0$ and $\alpha' = 2.0$, $\chi_1 = \chi_2 = \chi_3 = 1.0$ is taken and the dispersion curve is obtained, this curve in comparison with the dispersion curve of elliptic cylinder for the aspect ratio $a/b = 1.0$ and for the parameter $\alpha' = 2.0$, $\chi_1 = \chi_2 = \chi_3 = 1.0$ which represent a circle are same. The computed non-dimensional wavenumbers are presented as graphs.

5.3. Dispersion curves

The results of longitudinal and flexural (symmetric and antisymmetric) modes are plotted in figures with respect to different set of thermal parameters:

(i) $\alpha' = 2.0$, $\chi_1 = 2.6 \times 10^{-7}$, $\chi_2 = \chi_3 = 1.0$, (ii) $\alpha' = 2.0$, $\chi_1 = \chi_2 = \chi_3 = 1.0$, and (iii) $\alpha' = 4.0$, $\chi_1 = \chi_2 = \chi_3 = 1.0$. The notations LT, LWT represents the longitudinal mode with thermal field and longitudinal mode without thermal field, respectively. Similarly, FST, FSWT, FAT, and FAWT represent the flexural (symmetric and antisymmetric) modes with thermal and without thermal field in the order. The 1 refers to the first mode and 2 the second.

The comparison made between generalized thermal cylinder of arbitrary cross-section and a cylinder of arbitrary cross-section without thermal field in longitudinal and flexural (symmetric and antisymmetric) modes are, respectively, shown in Figs. 2 and 3. A graph is drawn between non-dimensional frequencies Ω versus dimensionless wavenumber $|\varsigma|$ for an elliptic cylinder for the longitudinal and flexural (symmetric and antisymmetric) modes of vibrations for the aspect ratios $a/b = 1.3$ with the thermal parameters $\alpha' = 2.0$, $\chi_1 = \chi_2 = \chi_3 = 1.0$ is given in Figs. 2 and 3, respectively. From the Fig. 2, it is observed that the wave propagation in thermal field dissipates more energy in the first mode of longitudinal vibration with respect to the second mode. The behavior of the wave is almost same for both the modes. The wave propagation in the cylinder without thermal field, the behavior is linear in the first mode, whereas in the second mode, there is a sudden rise in the wavenumber for small change in the frequency beyond which, a constant linearity is seen. It is observed in the Fig. 3, the general trend is, as the frequency Ω increases, the non-dimensional wavenumber $|\varsigma|$ also increases in both modes of vibration. In mode1, the behavior is same up to $\Omega = 0.3$, beyond which the increase in wavenumber of symmetric and antisymmetric without thermal field is slightly higher than the symmetric and antisymmetric vibration with thermal field. In mode 2, the antisymmetric with and without thermal field shows similar characteristics and symmetric mode with and without thermal field is same in behavior.

Aspect ratios $a/b = 1.5, 2.0$ versus dimensionless wavenumber $|\varsigma|$ of an elliptic cylinder for longitudinal and flexural (symmetric and antisymmetric) modes of vibrations with the parameters $\alpha' = 2.0$, $\chi_1 = \chi_2 = \chi_3 = 1.0$ are presented in Fig. 4. From the figure, it is observed that as the aspect ratios increase in the longitudinal and flexural (symmetric and antisymmetric) modes of vibration, the non-dimensional wavenumbers are also increases. Comparing the arbitrary cross-section with the circular cross-section (Fig. 1), the non-dimensional wavenumbers of the arbitrary cross-section is slightly more than the circular cross-section. The longitudinal

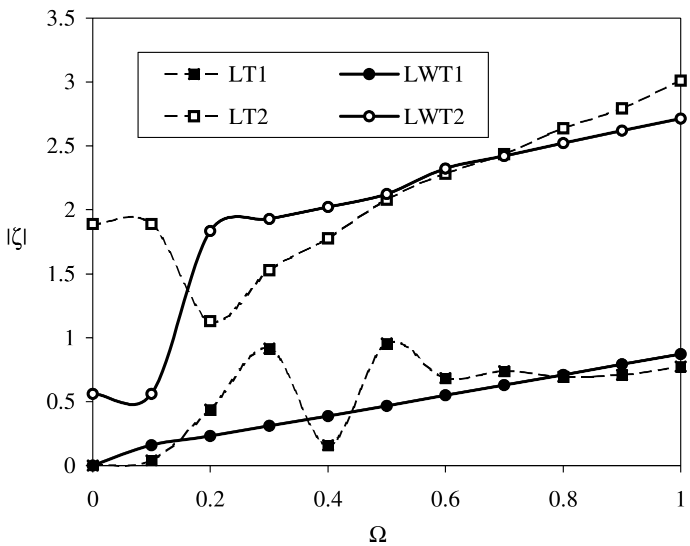

Fig. 2. Comparison between longitudinal mode of vibration in elliptic cross-sectional cylinder with and without thermal field for the parameters $\alpha' = 2.0$, $\chi_1 = 2.\ 6 \times 10^{-7}$, $\chi_2 = \chi_3 = 1.0$ for the aspect ratio $a/b = 1.3$.

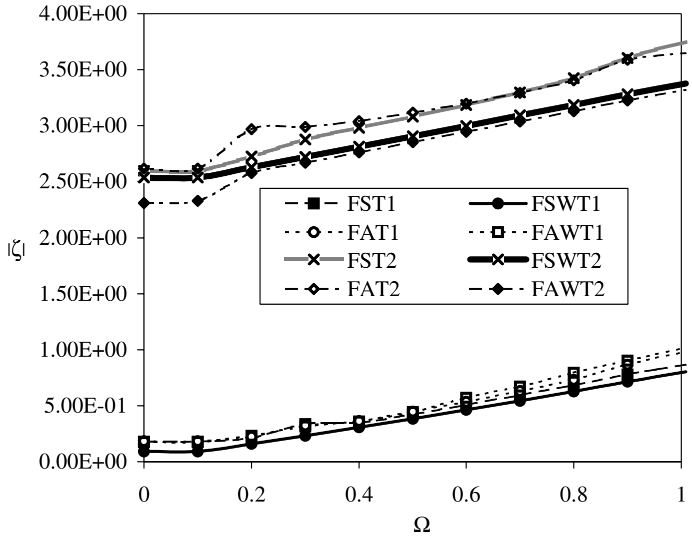

Fig. 3. Comparison between flexural (symmetric and antisymmetric) modes of vibration in elliptic cross-sectional cylinder with and without thermal field for the parameters $\alpha' = 2.0$, $\chi_1 = 2.6 \times 10^{-7}$, $\chi_2 = \chi_3 = 1.0$ for the aspect ratio $a/b = 1.3$.

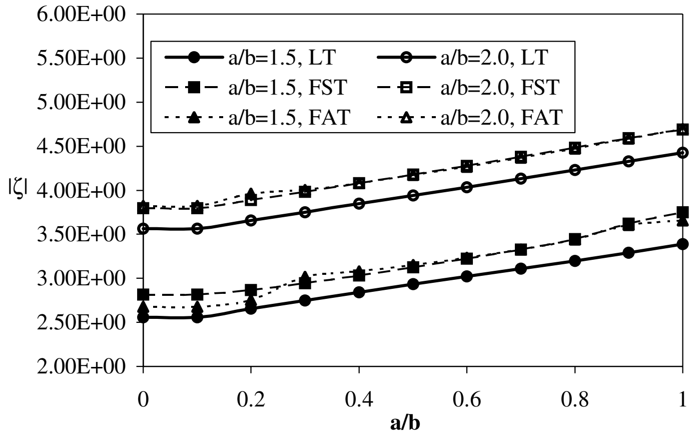

Fig. 4. Aspect ratios a/b versus dimensionless wavenumber $|\varsigma|$ of an elliptic cylinder for the longitudinal and flexural (symmetric and antisymmetric) modes of vibrations with $\alpha' = 2.0$, $\chi_1 = 2.6 \times 10^{-7}$, $\chi_2 = \chi_3 = 1.0$.

modes for the aspect ratios $a/b = 1.5, 2.0$ behave in the similar fashion. Similarly, the flexural (symmetric and antisymmetric) modes for the above aspect ratios behave same.

The Fig. 5 shows the non-dimensional frequency Ω versus dimensionless wavenumber $|\varsigma|$ for cardioidal cross-sectional cylinder with the material parameter $\alpha' = 2.0$, $\chi_1 = 2.6 \times 10^{-7}$, $\chi_2 = \chi_3 = 1.0$. By changing the parameter α', such as $\alpha' = 4.0$, $\chi_1 = 2.6 \times 10^{-7}$, $\chi_2 = \chi_3 = 1.0$, the dispersion curves are drawn and are presented in Fig. 6. From these figures, it is observed that as the frequency increases the non-dimensional wavenumbers are also increases. As comparing the vibration of circular cross-section $s = 0.0$ and the cardioid cross-section $s = 0.5$, the dispersion is high for cardioidal cross-section, than the circular cross-section, it is seen in Figs. 5 and 6.

6. Conclusions

In this paper, the wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section is analyzed by satisfying the boundary conditions on the irregular boundary using the Fourier expansion collo-

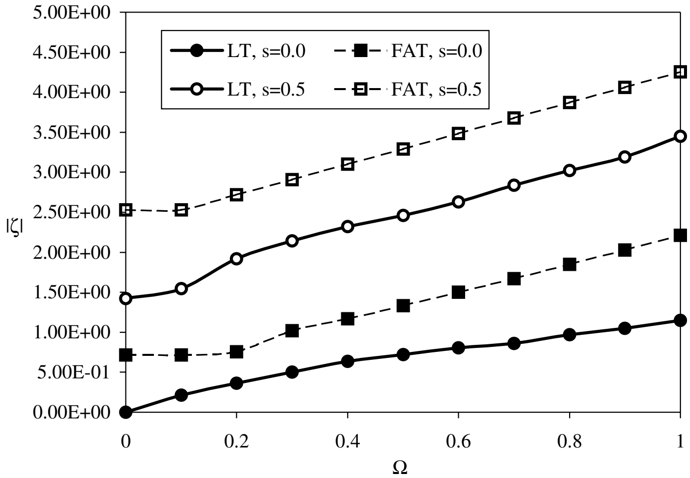

Fig. 5. Non-dimensional frequency Ω versus dimensionless wavenumber $|\varsigma|$ of a cardioidal cross-sectional cylinder for the longitudinal and flexural (antisymmetric) modes of vibrations with $\alpha' = 2.0$, $\chi_1 = 2.6 \times 10^{-7}$, $\chi_2 = \chi_3 = 1.0$.

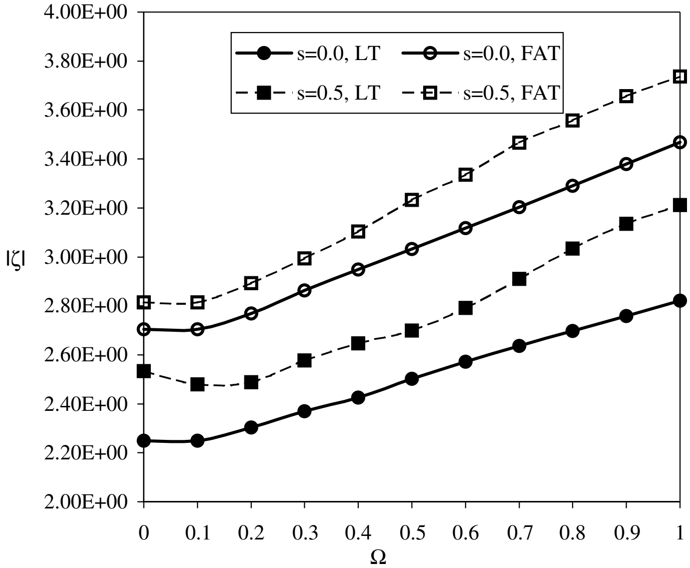

Fig. 6. Non-dimensional frequency Ω versus dimensionless wavenumber $|\varsigma|$ of a cardioidal cross-sectional cylinder for the longitudinal and flexural (antisymmetric) modes of vibrations with $\alpha' = 4.0$, $\chi_1 = 2.6 \times 10^{-7}$, $\chi_2 = \chi_3 = 1.0$.

cation method and the frequency equation for the longitudinal and flexural vibrations are obtained. Numerically the frequency equations are analyzed for the cylinder of different cross-section such as elliptical and cardioidal cross-sectional cylinders. The computed dimensionless wavenumbers of longitudinal and flexural (symmetric and antisymmetric) modes for the elliptic and cardioidal cross-sectional cylinders are plotted as dispersion curves. The results of the present method is compared with the literature results, they show very good agreement. The problem can be analyzed for any other cross-section by using the proper geometric relation.

Appendix A

The expressions $e_n^j \sim \bar{k}_n^j$ used in Eqs. (20) and (22) are given as follows:

$$\begin{aligned} e_n^j &= 2\varepsilon_4\{n(n-1)J_n(\alpha_j ax) + (\alpha_j ax)J_{n+1}(\alpha_j ax)\}\cos 2(\theta-\gamma_l)\cos n\theta \\ &\quad - x^2\left\{\varepsilon_4(\alpha_j a)^2 + \left[\bar{\lambda} + 2\cos^2(\theta-\gamma_l)\right] + \bar{\lambda} d_j \varepsilon_4 \varsigma\right\}J_n(\alpha_j ax)\cos n\theta \\ &\quad + 2n\varepsilon_4\{(n-1)J_n(\alpha_j ax) - (\alpha_j ax)J_{n+1}(\alpha_j ax)\}\sin n\theta \sin 2(\theta-\gamma_l), \quad j=1,2,3 \end{aligned} \tag{A.1}$$

$$\begin{aligned} e_n^4 &= 2\varepsilon_4\{n(n-1)J_n(\alpha_4 ax) - (\alpha_4 ax)J_{n+1}(\alpha_4 ax)\}\cos n\theta\cos 2(\theta-\gamma_l) \\ &\quad + 2\varepsilon_4\left\{[n(n-1) - (\alpha_4 ax)^2]J_n(\alpha_4 ax) + (\alpha_4 ax)J_{n+1}(\alpha_4 ax)\right\}\sin n\theta\sin 2(\theta-\gamma_l) \end{aligned} \tag{A.2}$$

$$\begin{aligned} f_n^j &= 2\left\{[n(n-1) - (\alpha_j ax)^2]J_n(\alpha_j ax) + (\alpha_j ax)J_{n+1}(\alpha_j ax)\right\}\cos n\theta\sin 2(\theta-\gamma_l) \\ &\quad + 2n\{(\alpha_j ax)J_{n+1}(\alpha_j ax) - (n-1)J_n(\alpha_j ax)\}\sin n\theta\cos 2(\theta-\gamma_l), \quad j=1,2,3 \end{aligned} \tag{A.3}$$

$$\begin{aligned} f_n^4 &= 2n\varepsilon_4\{(n-1)J_n(\alpha_4 ax) - (\alpha_4 ax)J_{n+1}(\alpha_4 ax)\}\cos n\theta\sin 2(\theta-\gamma_l) \\ &\quad - \varepsilon_4\left\{2(\alpha_4 ax)J_{n+1}(\alpha_4 ax) - [(\alpha_4 ax)^2 - 2n(n-1)]J_n(\alpha_4 ax)\right\}\sin n\theta\cos 2(\theta-\gamma_l) \end{aligned} \tag{A.4}$$

$$g_n^j = (\varsigma + d_j)\{nJ_n(\alpha_j ax)\cos(\overline{n-1}\theta + \gamma_l) - (\alpha_j ax)J_{n+1}(\alpha_j ax)\cos(\theta-\gamma_l)\cos n\theta\}, \quad j=1,2,3 \tag{A.5}$$

$$g_n^4 = \varsigma\{nJ_n(\alpha_4 ax)\cos(\overline{n-1}\theta + \gamma_l) - (\alpha_4 ax)J_{n+1}(\alpha_4 ax)\sin n\theta\sin(\theta-\gamma_l)\} \tag{A.6}$$

$$k_n^j = e_j\{n\cos(\overline{n-1}\theta + \gamma_l)J_n(\alpha_j ax) - (\alpha_j ax)J_{n+1}(\alpha_j ax)\cos(\theta-\gamma_l)\cos n\theta\}, \quad j=1,2,3 \tag{A.7}$$

$$\begin{aligned} \bar{e}_n^j &= 2\varepsilon_4\{n(n-1)J_n(\alpha_j ax) + (\alpha_j ax)J_{n+1}(\alpha_j ax)\}\cos 2(\theta-\gamma_l)\sin n\theta \\ &\quad - x^2\left\{\varepsilon_4(\alpha_j a)^2 + \left[\bar{\lambda} + 2\cos^2(\theta-\gamma_l)\right] + \bar{\lambda} d_j \varepsilon_4 \varsigma\right\}J_n(\alpha_j ax)\cos n\theta \\ &\quad - 2n\varepsilon_4\{(n-1)J_n(\alpha_j ax) - (\alpha_j ax)J_{n+1}(\alpha_j ax)\}\cos n\theta\sin 2(\theta-\gamma_l), \quad j=1,2,3 \end{aligned} \tag{A.8}$$

$$\begin{aligned} \bar{e}_n^4 &= 2\varepsilon_4\{n(n-1)J_n(\alpha_4 ax) - (\alpha_4 ax)J_{n+1}(\alpha_4 ax)\}\sin n\theta\cos 2(\theta-\gamma_l) \\ &\quad - 2\varepsilon_4\left\{[n(n-1) - (\alpha_4 ax)^2]J_n(\alpha_4 ax) + (\alpha_4 ax)J_{n+1}(\alpha_4 ax)\right\}\cos n\theta\sin 2(\theta-\gamma_l) \end{aligned} \tag{A.9}$$

$$\begin{aligned} \bar{f}_n^j &= 2\left\{[n(n-1) - (\alpha_j ax)^2]J_n(\alpha_j ax) + (\alpha_j ax)J_{n+1}(\alpha_j ax)\right\}\sin n\theta\sin 2(\theta-\gamma_l) \\ &\quad - 2n\{(\alpha_j ax)J_{n+1}(\alpha_j ax) - (n-1)J_n(\alpha_j ax)\}\cos n\theta\cos 2(\theta-\gamma_l), \quad j=1,2,3 \end{aligned} \tag{A.10}$$

$$\begin{aligned} \bar{f}_n^4 &= 2n\varepsilon_4\{(n-1)J_n(\alpha_4 ax) - (\alpha_4 ax)J_{n+1}(\alpha_4 ax)\}\sin n\theta\sin 2(\theta-\gamma_l) \\ &\quad + \varepsilon_4\left\{2(\alpha_4 ax)J_{n+1}(\alpha_4 ax) - [(\alpha_4 ax)^2 - 2n(n-1)]J_n(\alpha_4 ax)\right\}\cos n\theta\cos 2(\theta-\gamma_l) \end{aligned} \tag{A.11}$$

$$\bar{g}_n^j = (\varsigma + d_j)\{nJ_n(\alpha_j ax)\cos(\overline{n-1}\theta + \gamma_l) - (\alpha_j ax)J_{n+1}(\alpha_j ax)\cos(\theta-\gamma_l)\sin n\theta\}, \quad j=1,2,3 \tag{A.12}$$

$$\bar{g}_n^4 = \varsigma\{nJ_n(\alpha_4 ax)\cos(\overline{n-1}\theta + \gamma_l) + (\alpha_4 ax)J_{n+1}(\alpha_4 ax)\cos n\theta\sin(\theta-\gamma_l)\} \tag{A.13}$$

$$\bar{k}_n^j = e_j\{n\cos(\overline{n-1}\theta + \gamma_l)J_n(\alpha_j ax) + (\alpha_j ax)J_{n+1}(\alpha_j ax)\cos(\theta-\gamma_l)\sin n\theta\}, \quad j=1,2,3 \tag{A.14}$$

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